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Experimental Restrictions on Spin–Spin Interaction in Gauge Gravity

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The article deals with the Poincaré gauge theory of gravity with the most general Lagrangian quadratic in curvature and torsion. We consider three special cases of this model. Two effects are calculated within these models: the hyperfine energy level splitting of the hydrogen atom and the interaction between polarized photons in a sodium vapor. We find that none of these models allows one to estimate model constants consistently.

1. MODEL OF THE GAUGE THEORY OF GRAVITY

This article continues the work of Obukhov and Yakushin (1991), where we considered the Poincaré gauge theory of gravity with the quadratic Lagrangian

$$
\begin{aligned}
L_g = -\frac{1}{16\pi G}(&\tilde{R} - 2\Lambda + b_1 \tilde{R}_{\alpha\beta\mu\nu}\tilde{R}^{\alpha\beta\mu\nu} + b_2 \tilde{R}_{\alpha\beta\mu\nu}\tilde{R}^{\mu\nu\alpha\beta} + b_3 \tilde{R}_{\alpha\beta\mu\nu}\tilde{R}^{\alpha\mu\beta\nu} \\
&+ b_4 \tilde{R}_{\alpha\beta}\tilde{R}^{\alpha\beta} + b_5 \tilde{R}_{\alpha\beta}\tilde{R}^{\beta\alpha} + b_6 \tilde{R}^2 + a_1 Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} + a_2 Q_{\alpha\mu\nu}Q^{\mu\alpha\nu} \\
&+ a_3 Q_\mu Q^\mu + b_7 \tilde{R}_{\alpha\beta}D^{\alpha\beta} + b_8 \tilde{R}_{\alpha\beta}D^{\beta\alpha} + b_9 \tilde{R}D \\
&+ a_4 Q_{\alpha\beta\mu}Q^{*\alpha\beta\mu} + a_5 Q_\mu P^\mu)
\end{aligned} \tag{1}
$$

where $\tilde{R}^\alpha{}_{\beta\mu\nu}$, $\tilde{R}_{\mu\nu}$, and $\tilde{R}$ are the Riemann–Cartan curvature tensor and its contractions; $Q^\alpha{}_{\mu\nu}$ is the torsion; $Q^{*\alpha}{}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}Q^{\alpha\rho\sigma}$; $D_{\mu\nu} = \frac{1}{2}\varepsilon^{\alpha\beta\lambda}{}_\mu \tilde{R}_{\alpha\beta\lambda\nu}$; $D = D_{\mu\nu}g^{\mu\nu}$; Λ is the cosmological constant; and a_i, b_i are the coupling constants.

The torsion tensor can be decomposed into three irreducible parts: $Q^\alpha{}_{\mu\nu} = C^\alpha{}_{\mu\nu} + T^\alpha{}_{\mu\nu} + P^\alpha{}_{\mu\nu}$, where $C^\alpha{}_{\mu\nu}$ is the traceless part of the torsion

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($C^\alpha{}_{\mu\alpha}=0$, $C^\alpha{}_{\mu\nu}=-C^\alpha{}_{\nu\mu}$, $C^\alpha{}_{\mu\nu}+C_{\mu\nu}{}^\alpha+C_\nu{}^\alpha{}_\mu=0$); $T^\alpha{}_{\mu\nu}=\frac{2}{3}\delta^\alpha_{[\nu}Q_{\mu]}$, $Q_\mu=Q^\nu{}_{\mu\nu}$ is the trace of the torsion; and $P^\alpha{}_{\mu\nu}=\frac{1}{3}\varepsilon^\alpha{}_{\mu\nu\lambda}P^\lambda$, is the pseudotrace of the torsion. This model has been considered in detail in Obukhov *et al.* (1989).

We are interested in microscopic effects in this model. Therefore we assume $g_{\mu\nu}=\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-1,-1)$. Limiting ourselves to the tree approximation, we shall not take into account terms of self-interaction for the torsion fields.

In Obukhov and Yakushin (1990) the special case $P_\mu\neq0$, $Q_\mu=0$, $C^\alpha{}_{\mu\nu}=0$ was discussed. It is described by the following Lagrangian, to which (1) reduces under the given restrictions:

$$L=-\tfrac{1}{4}(\partial_\mu P'_\nu-\partial_\nu P'_\mu)^2+\tfrac{1}{2}\mu^2(P'_\mu)^2-\frac{\lambda}{2}(\partial P')^2 \tag{2}$$

where $P'_\mu=(1/2\chi)P_\mu$; hereafter all the constants which are not described in the text are given in the Appendix.

We shall consider here other special cases: (a) $P_\mu=0$, $Q_\mu\neq0$, $C^\alpha{}_{\mu\nu}=0$, with

$$L=-\tfrac{1}{4}(\partial_\mu Q'_\nu-\partial_\nu Q'_\mu)^2+\tfrac{1}{2}M^2(Q'_\mu)^2-\tfrac{1}{2}\lambda_1(\partial Q')^2 \tag{3}$$

where $Q'_\mu=(1/\varkappa)Q_\mu$; and (b) $P_\mu\neq0$, $Q_\mu\neq0$, $C^\alpha{}_{\mu\nu}=0$, with

$$\begin{aligned}L=&-\tfrac{1}{4}(\partial_\mu P'_\nu-\partial_\nu P'_\mu)^2-\tfrac{1}{2}\lambda(\partial P')^2+\tfrac{1}{2}\mu^2(P'_\mu)^2-\tfrac{1}{4}(\partial_\mu Q'_\nu-\partial_\nu Q'_\mu)^2\\&-\tfrac{1}{2}\lambda_1(\partial Q')^2+\tfrac{1}{2}M^2(Q'_\mu)^2+\frac{A}{4}(\partial_\mu P'_\nu-\partial_\nu P'_\mu)(\partial_\nu Q'_\mu-\partial_\mu Q'_\nu)\\&+\frac{B}{2}(\partial P')(\partial Q')-\frac{C}{2}(P'Q')\end{aligned} \tag{4a}$$

After the change of variables

$$P'_\mu=K_\mu+R_1M_\mu,\qquad Q'_\mu=\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}K_\mu+\frac{1+(1-4R_1R_2)^{1/2}}{2}M_\mu$$

where $R_1=(A\lambda_1-B)/2(\lambda_1-\lambda)$, $R_2=(B-A\lambda)/2(\lambda_1-\lambda)$, (4a) takes the form

$$\begin{aligned}L=&-\tfrac{1}{4}(\partial_\mu K'_\nu-\partial_\nu K'_\mu)^2-\tfrac{1}{2}a_1(\partial K')^2+\tfrac{1}{2}m_1^2(K'_\mu)^2-\tfrac{1}{4}(\partial_\mu M'_\nu-\partial_\nu M'_\mu)^2\\&-\tfrac{1}{2}a_2(\partial M')^2+\tfrac{1}{2}m_2^2(M'_\mu)^2+w(K'M')\end{aligned} \tag{4b}$$

where $K'_\nu=S_KK_\nu$, $M'_\nu=S_MM_\nu$.

2. HYPERFINE SPLITTING

The hyperfine splitting of energy levels of the hydrogen atom, estimated within the framework of QED, slightly differs from its experimental value (Hayashi and Sasaki, 1978)

$$\left|\frac{\Delta\nu_{\rm exp}-\Delta\nu_{\rm th}}{\Delta\nu_{\rm th}}\right|<7\times10^{-6} \tag{5}$$

Assuming that this difference is due to torsion interaction, in Obukhov and Yakushin (1991) we calculated the contribution of an axial torsion described by (2) to the hyperfine splitting of the $1S$ state of the hydrogen atom:

$$\Delta E=-\frac{4\chi^2(m\alpha)^3}{3\pi}\left(\frac{2}{(\mu+2m\alpha)^2}+\frac{1}{(\mu+2\lambda^{1/2}m\alpha)^2}\right) \tag{6}$$

where m is the mass of the electron, and α is the fine structure constant.

In the case (3) the vector part of the torsion does not conribute to the effect because it does not interact with the spinor field.

However, in the model (4) the field Q_μ contributes to the effect due to the interaction with P_μ. The total Lagrangian of interacting spinor, vector, and pseudovector fields reads

$$\begin{aligned} L=&-\tfrac{1}{4}(\partial_\mu K'_\nu-\partial_\nu K'_\mu)^2-\tfrac{1}{2}a_1(\partial K')^2+\tfrac{1}{2}m_1^2(K'_\mu)^2-\tfrac{1}{4}(\partial_\mu M'_\nu-\partial_\nu M'_\mu)^2\\ &-\tfrac{1}{2}a_2(\partial M')^2+\tfrac{1}{2}m_2^2(M'_\mu)^2+w(K'M')+\frac{i}{2}[\bar\psi\gamma_\mu\partial_\mu\psi-\partial_\mu\bar\psi\gamma_\mu\psi]\\ &-M_\psi\bar\psi\psi-\chi_K j_{5\lambda}K'_\lambda-\chi_M j_{5\lambda}M'_\lambda \end{aligned} \tag{7}$$

where $j_{5\rho}=\bar\psi\gamma_5\gamma_\rho\psi$, $\chi_K=\chi/S_K$, and $\chi_M=\chi R_1/S_M$.

The relevant equations of motion for torsion fields are as follows:

$$\begin{aligned} (\Box+m_1^2)K'_\mu-(1-a_1)\partial_\mu(\partial K')&=\chi_K j_{5\mu}-wM'_\mu\\ (\Box+m_2^2)M'_\mu-(1-a_2)\partial_\mu(\partial M')&=\chi_M j_{5\mu}-wK'_\mu \end{aligned} \tag{8}$$

The propagators of fields K'_μ and M'_μ in the nonrelativistic approximation are

$$\begin{aligned} G^K_{nm}=&\frac{\mathbf{k}^2+m_2^2-\chi_M w/\chi_K}{(\mathbf{k}^2+m_1^2)(\mathbf{k}^2+m_2^2)-w^2}\left(\delta_{nm}-\frac{k_nk_m}{\mathbf{k}^2}\right)\\ &+\frac{(a_2\mathbf{k}^2+m_2^2-w\chi_M/\chi_K)k_nk_m}{[(a_1\mathbf{k}^2+m_1^2)(a_2\mathbf{k}^2+m_2^2)-w^2]\mathbf{k}^2}\\ G^M_{nm}=&G^K_{nm}(m_1\leftrightarrow m_2,\ \chi_K\leftrightarrow\chi_M) \end{aligned} \tag{9}$$

The nonrelativistic limit of spinor currents is $\bar{u}'\gamma_5\gamma_0 u = -u'^{+}\gamma_5 u = 0$, $\bar{u}'\gamma_5\gamma^i u = -2mW'^{+}\sigma_i W$, where

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

W is a nonrelativistic two-component spinor normalized by the condition $WW^+ = 1$. Repeating speculations of Obukhov and Yakushin (1991), we can find spherically symmetric static potentials and hyperfine splittings. There are three cases:

Case 1. $w^2 < m_1^2 m_2^2$. Here

$$V_1^s(r) = \frac{\boldsymbol{\sigma}^{(e)}\boldsymbol{\sigma}^{(p)}}{r}(A_1 e^{-C_1 r} + A_2 e^{-C_2 r} + A_3 e^{-C_3 r} + A_4 e^{-C_4 r})$$
$$\Delta E_1 = 16(m\alpha)^3 \sum_{i=1}^{4} \frac{A_i}{(C_i + 2m\alpha)^2} \tag{10a}$$

Case 2. $w^2 = m_1^2 m_2^2$. Here

$$V_2^s(r) = \frac{\boldsymbol{\sigma}^{(e)}\boldsymbol{\sigma}^{(p)}}{r}(B_1 e^{-L_1 r} + B_2 e^{-L_2 r} + B_3)$$
$$\Delta E_2 = 16(m\alpha)^3 \left[\frac{B_1}{(L_1 + 2m\alpha)^2} + \frac{B_2}{(L_2 + 2m\alpha)^2} + \frac{B_3}{(2m\alpha)^2}\right] \tag{10b}$$

Case 3. $w^2 > m_1^2 m_2^2$. Here

$$V_3^s(r) = \frac{\boldsymbol{\sigma}^{(e)}\boldsymbol{\sigma}^{(p)}}{r} \times [D_1 e^{-C_1 r} + D_2 \cos(C_2' r) + D_3 e^{-C_3 r} + D_4 \cos(C_4' r)]$$
$$\Delta E_3 = 16(m\alpha)^3 \left[\frac{D_1}{(C_1 + 2m\alpha)^2} + D_2 \frac{(2m\alpha)^2 - C_2'^2}{(2m\alpha)^2 + C_2'^2} + \frac{D_3}{(C_3 + 2m\alpha)^2} + D_4 \frac{(2m\alpha)^2 - C_4'^2}{(2m\alpha)^2 + C_4'^2}\right] \tag{10c}$$

In fact these three cases are not independent. Supposing $w^2 = m_1^2 m_2^2 - \varepsilon^2$, we obtain $C_1 \to L_1 + O(\varepsilon)$, $C_2 \to \varepsilon/L_1 + O(\varepsilon^2)$, $C_3 \to L_2 + O(\varepsilon)$, $C_4 \to \varepsilon/[L_2(a_1 a_2)^{1/2}] + O(\varepsilon^2)$; $V_1(r) \to V_2(r)$, $E_1 \to E_2$. Analogously, we can go from case 3 to case 2. One can obtain case 3 from case 1 as follows:

$$\frac{1}{2}\left\{\begin{bmatrix} V_1(r) \\ E_1 \end{bmatrix}(C_{2,4}\to iC'_{2,4})+\begin{bmatrix} V_1(r) \\ E_1 \end{bmatrix}(C_{2,4}\to -iC'_{2,4})\right\}=\begin{bmatrix} V_3(r) \\ E_3 \end{bmatrix}$$

The same approach leads from case 3 to case 1. If we switch off the vector–pseudovector interaction, we are left with the results of Obukhov and Yakushin (1991) in the limit.

3. INTERACTION OF POLARIZED PHOTONS

Tam and Happer (1977*a,b*) discovered the following effect. Two laser beams of opposite (equal) polarizations with a frequency slightly higher than that of the D_1 line in the sodium atom spectrum repel (attract) each other in the medium of sodium vapor. Naik and Pradhan (1981) suggested that the interaction between photons is mediated by an axial torsion, and the enhancement of this effect in sodium vapor is due to an induced polarization of the medium. They took the Lagrangian to be of the electrodynamic type for the axial vector torsion. We, Obukhov and Yakushin (1991), considered this effect within the model (2) and found

$$\boldsymbol{\theta}=\boldsymbol{\theta}_s-\boldsymbol{\theta}_0=\frac{\Omega}{|\mathbf{k}|}\left(1-\frac{\mu^2}{\Omega^2}+\frac{32n\chi^2}{9\Omega^3}\right)^{1/2}-\boldsymbol{\theta}_0 \tag{11}$$

Here $\boldsymbol{\theta}$ is the angle between the incident and scattered beams, n is the concentration of sodium atoms, $\Omega=17\ \mathrm{cm}^{-1}$ is the difference of energy levels D_1 and D_2, $|\mathbf{k}|=16970\ \mathrm{cm}^{-1}$ is the momentum of incident photons, and $2\boldsymbol{\theta}_0=2$ mrad is the angle of the initial convergence of the beams. Tam and Happer measured $\boldsymbol{\theta}(n)$. Comparing the experimental data with (11), we obtained

$$\mu^2\approx 0, \qquad \chi^2\approx 5\times 10^{-9} \tag{12}$$

It is convenient to assume that μ is very small, but not zero. The correct sign of the formally calculated two-body potential of the photon–photon interaction is provided by $\lambda=1$. Under these conditions we have from (6) $\Delta E=-\chi^2 m\alpha/\pi$. Hence

$$\chi^2<6\times 10^{-15} \tag{13}$$

The estimates (12) and (13) are mutually inconsistent. So let us discuss cases (a) and (b) of Section 1 in an effort to escape this inconsistency.

We introduce the interaction between torsion and electromagnetic fields as in Naik and Pradhan (1981). Then one finds for a vector torsion

$$L=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+\tfrac{2}{3}\varkappa(\partial_\mu A_\nu-\partial_\nu A_\mu)A_\mu Q'_\nu$$
$$-\tfrac{1}{4}(\partial_\mu Q'_\nu-\partial_\nu Q'_\mu)^2+\tfrac{1}{2}M^2(Q'_\mu)^2-\frac{\lambda_1}{2}(\partial Q')^2 \qquad (14)$$

and for interacting vector and pseudovector fields

$$L=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+\chi_K^{(1)}K'_\lambda\varepsilon_{\alpha\nu\mu\lambda}A_\alpha(\partial_\mu A_\nu-\partial_\nu A_\mu)$$
$$+\chi_M^{(1)}M'_\lambda\varepsilon_{\alpha\nu\mu\lambda}A_\alpha(\partial_\mu A_\nu-\partial_\nu A_\mu)+\chi_K^{(2)}K'_\nu A_\mu(\partial_\mu A_\nu-\partial_\nu A_\mu)$$
$$+\chi_M^{(2)}M'_\nu A_\mu(\partial_\mu A_\nu-\partial_\nu A_\mu)-\tfrac{1}{4}(\partial_\mu K'_\nu-\partial_\nu K'_\mu)^2$$
$$-\tfrac{1}{2}a_1(\partial K')^2+\tfrac{1}{2}m_1^2(K'_\mu)^2-\tfrac{1}{4}(\partial_\mu M'_\nu-\partial_\nu M'_\mu)^2-\tfrac{1}{2}a_2(\partial M')^2$$
$$+\tfrac{1}{2}m_2^2(M'_\mu)^2+w(K'M') \qquad (15)$$

where

$$\chi_K^{(1)}=\frac{2\chi}{3S_K},\qquad \chi_M^{(1)}=\frac{2\chi R_1}{3S_M}$$

$$\chi_K^{(2)}=\frac{\varkappa(1-4R_1R_2)^{1/2}}{3R_1S_K},\qquad \chi_M^{(2)}=\frac{\varkappa[1+(1-4R_1R_2)^{1/2}]}{3S_M}$$

Following the lines of Naik and Pradhan (1981), one obtains that the pure vector field does not contribute to the phenomenon.

Let us consider the model (15). In this case $\boldsymbol{\theta}=(\mathbf{q}^2/\mathbf{k}^2)^{1/2}-\boldsymbol{\theta}_0$, where

$$\mathbf{q}^2=\Omega^2+\tfrac{1}{2}[-(m_1^2+m_2^2)+s(\chi_K^{(1)2}+\chi_M^{(1)2})\pm\sqrt{D}]$$

$$s=\frac{8n}{\Omega}$$

$$D=(m_1^2-m_2^2)^2+4w^2+s^2(\chi_K^{(1)2}+\chi_M^{(1)2})^2+2s(m_2\chi_K^{(1)}-m_1\chi_M^{(1)})^2$$
$$-2s(\chi_K^{(1)}m_1+\chi_M^{(1)}m_2)^2$$

The experimental data are well approximated by $y=A_0x$ (Obukhov and Yakushin, 1991), where

$$y\equiv|\mathbf{k}|(\theta+\theta_0)^2-\Omega^2\equiv\mathbf{q}^2-\Omega^2,\qquad x\equiv n\times10^{-12},\qquad A_0=1050\text{ cm}$$

$$[\text{for (3)},\ A_0\equiv32\chi^2 10^{12}/9\Omega]$$

Supposing that in the model (5) the approximation remains the same or changes very little, one should impose the following conditions on the constants: $m_1^2 = m_2^2 = w = 0$. We, however, assume $m_i \neq 0$, but very small. The correct sign of the interaction is provided by $a_i = 1$. Then finally one gets $A_0 = 32(\chi_K^2 + \chi_M^2)10^{12}/9\Omega$ and $\Delta E = -(\chi_K^2 + \chi_M^2)m\alpha/\pi$. These results, of course, do not solve the problem of inconsistency.

What can this inconsistency be connected with? First, we have not taken into account the traceless part of the torsion. Second, the introduction of an interaction (Naik and Pradhan, 1981) between the electromagnetic and torsion fields may be incorrect. Third, some other mechanism perhaps can contribute to the Tam–Happer phenomenon besides the torsion one. These possibilities are under investigation and will be discussed elsewhere.

APPENDIX

$$\mu^2 = -\frac{3\mu_1}{\Lambda_5}, \qquad \lambda = \frac{3\Lambda_4}{2\Lambda_5}, \qquad \chi = \frac{3}{4}\left(\frac{16\pi G}{-\Lambda_5}\right)^{1/2}$$

Let $\Lambda_5 < 0$, $\Lambda_4 < 0$, $\mu_1 > 0$.

$$M^2 = \frac{3\mu_2}{2\Lambda_6}, \qquad \lambda_1 = \frac{3\Lambda_3}{2\Lambda_6}, \qquad \varkappa = \frac{3}{4}\left(\frac{16\pi G}{\Lambda_6}\right)^{1/2}$$

Let $\Lambda_6 > 0$, $\mu_2 > 0$, $\Lambda_3 > 0$.

$$A = \frac{\Lambda_7}{2(-\Lambda_5\Lambda_6)^{1/2}}, \qquad B = \frac{3\Lambda_8}{(-\Lambda_5\Lambda_6)^{1/2}}, \qquad C = \frac{9\mu_4}{4(-\Lambda_5\Lambda_6)^{1/2}}$$

$$\Lambda_3 = 4(b_1 + b_2) + 2b_3 + 4(b_4 + b_5) + 12b_6$$

$$\Lambda_4 = 4b_1 - 2b_3$$

$$\Lambda_5 = 4(b_1 - b_2) + b_4 - b_5$$

$$\Lambda_6 = 4b_1 + b_3 + 2b_4$$

$$\Lambda_7 = 6b_7 - 2b_8$$

$$\Lambda_8 = 2(b_7 + b_8) + 6b_9$$

$$\mu_1 = a_1 - a_2 - 1,$$

$$\mu_2 = 2 - \frac{2a_1 + a_2 + 3a_3}{4},$$

$$\mu_4 = \tfrac{4}{3}a_4 + a_5$$

For the details about Λ_i and μ_i see Obukhov and Yakushin (1991) and Obukhov *et al.* (1989).

$$S_K^2=1+\left(\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}\right)^2-A\,\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}$$

$$S_M^2=R_1^2+\left(\frac{1+(1-4R_1R_2)^{1/2}}{2}\right)^2-AR_1\,\frac{1+(1-4R_1R_2)^{1/2}}{2}$$

$$m_1^2=\left[\mu^2+M^2\left(\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}\right)^2-C\,\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}\right]\Big/S_K^2$$

$$m_2^2=\left[\mu^2R_1^2+M^2\left(\frac{1+(1-4R_1R_2)^{1/2}}{2}\right)^2-CR_1\,\frac{1+(1-4R_1R_2)^{1/2}}{2}\right]\Big/S_M^2$$

$$a_1=\left[\lambda+\lambda_1\left(\frac{1-(1-4R_1R_2)^{1/2}}{2R_1}\right)^2-B\,\frac{(1-4R_1R_2)^{1/2}}{2R_1}\right]\Big/S_K^2$$

$$a_2=\left[\lambda R_1^2+\lambda_1\left(\frac{1+(1-4R_1R_2)^{1/2}}{2}\right)^2-BR_1\,\frac{1+(1-4R_1R_2)^{1/2}}{2}\right]\Big/S_M^2$$

$$w=\frac{2R_1\mu^2+2R_2M^2-C}{2S_KS_M}$$

$$C_1^2=\tfrac{1}{2}\{m_1^2+m_2^2+[(m_1^2-m_2^2)^2+4w^2]^{1/2}\}$$

$$C_2^2=\tfrac{1}{2}\{m_1^2+m_2^2-[(m_1^2-m_2^2)^2+4w^2]^{1/2}\}$$

$$C_3^2=\tfrac{1}{2}\{m_1^2/a_1+m_2^2/a_2+[(m_1^2/a_1-m_2^2/a_2)^2+4w^2/(a_1a_2)]^{1/2}\}$$

$$C_4^2=\tfrac{1}{2}\{m_1^2/a_1+m_2^2/a_2-[(m_1^2/a_1-m_2^2/a_2)^2+4w^2/(a_1a_2)]^{1/2}\}$$

$$C_2'^2=-\tfrac{1}{2}\{m_1^2+m_2^2-[(m_1^2-m_2^2)^2+4w^2]^{1/2}\}$$

$$C_4'^2=-\tfrac{1}{2}\{m_1^2/a_1+m_2^2/a_2-[(m_1^2/a_1-m_2^2/a_2)^2+4w^2/(a_1a_2)]^{1/2}\}$$

$$L_1^2=m_1^2+m_2^2,\qquad L_2^2=m_1^2/a_1+m_2^2/a_2$$

$$A_1=\frac{\chi_K^2(C_1^2-m_2^2)+\chi_M^2(C_1^2-m_1^2)+2w\chi_K\chi_M}{6\pi(C_2^2-C_1^2)}$$

$$A_2=\frac{\chi_K^2(C_2^2-m_2^2)+\chi_M^2(C_2^2-m_1^2)+2w\chi_K\chi_M}{6\pi(C_1^2-C_2^2)}$$

$$A_3=\frac{\chi_K^2(C_3^2-m_2^2/a_2)+\chi_M^2(C_3^2-m_1^2/a_1)+2w\chi_K\chi_M/(a_1a_2)}{12\pi(C_4^2-C_3^2)}$$

$$A_4=\frac{\chi_K^2(C_4^2-m_2^2/a_2)+\chi_M^2(C_4^2-m_1^2/a_1)+2w\chi_K\chi_M/(a_1a_2)}{12\pi(C_3^2-C_4^2)}$$

$$B_1 = -\frac{(m_1\chi_K + m_2\chi_M)^2}{6\pi L_1^2}$$

$$B_2 = -\frac{(\chi_K m_1/a_1 + \chi_M m_2/a_2)^2}{12\pi L_1^2}$$

$$B_3 = -\frac{(\chi_K m_2 - \chi_M m_1)^2}{12\pi L_1^2}\left(2 + \frac{L_1^2}{a_1 a_2 L_2^2}\right)$$

$$D_1 = -\frac{\chi_K^2(C_1^2 - m_2^2) + \chi_M^2(C_1^2 - m_1^2) + 2w\chi_K\chi_M}{6\pi(C_2'^2 + C_1^2)}$$

$$D_2 = -\frac{\chi_K^2(C_2'^2 + m_2^2) + \chi_M^2(C_2'^2 + m_1^2) - 2w\chi_K\chi_M}{6\pi(C_1^2 + C_2'^2)}$$

$$D_3 = -\frac{\chi_K^2(C_3^2 - m_2^2/a_2) + \chi_M^2(C_3^2 - m_1^2/a_1) + 2w\chi_K\chi_M/(a_1a_2)}{12\pi(C_4'^2 + C_3^2)}$$

$$D_4 = -\frac{\chi_K^2(C_4'^2 + m_2^2/a_2) + \chi_M^2(C_4'^2 + m_1^2/a_1) + 2w\chi_K\chi_M/(a_1a_2)}{12\pi(C_3^2 + C_4'^2)}$$

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